ELEMENTARY THEORY OF THE BOILING LAYER

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This study is an attempt at a qualitative examination of the phenomena of a boiling layer, based on certain probable hypotheses and estimates. It is not suggested that the results obtained be considered final. Many studies have been dedicated to the problems of the boiling layer. An incomplete bibliography is contained in [1]. But at present there is no physical theory of the boiling layer which would describe the fundamental principles of the phenomena observed, even though many studies have been directed in this direction, for example [2]. The main difficulty encountered is that the boiling layer is a dissipative system, with the mechanism which generates chaotic particle motion unknown. There are no conservation integrals such as the energy integral, and thus a formulation analogous to the kinetic theory of gases is inapplicable.

1. Basic Facts. We will imagine a vertical cylindrical tube, in the interior of which is a layer of identical globules, resting on a grid. Through the tube from bottom to top there passes a flow of an incompressible fluid, for example, air or water. This flow produces a lifting force acting upon the layer of particles opposite to the force of gravity. When the lifting force becomes equal to the gravitational force the layer becomes, so to speak, "weightless." With a further increase in flow velocity a situation is possible in which the layer, not changing its structure, may begin to move upward, like a piston. However, in practice this does not occur; the layer expands, the particle concentration therein decreases, the interparticle distance increases, and so does the flow velocity in the layer, and thus the lifting force decreases. Within the layer there develops a particle configuration such that the lifting force again equals the gravitational force.

Observations have shown that in a boiling layer various regimes are possible, depending on the flow rate and properties of the medium and particles. The most basic of these modes is the so-called homogeneous boiling layer. In this case the flow in the layer is distributed almost uniformly over the cross-section, almost the entire particle mass is concentrated in a column of definite height, possessing a well defined surface, above which exists "vapor," where the particle concentration is significantly lower than in the layer. Not infrequently one can observe oscillations and waves on the layer surface similar to those on the surface of water, whence the boiling layer is also called the quasi-liquid layer. The particle concentration in a sufficiently thick column has practically no variation with height; in the vapor phase the concentration decreases rapidly with height. Thus the problem of a satisfactory theory is complicated by the fact that it must describe a phase transition, and the differential equation describing particle concentration as a function of height must allow discontinuous solutions.

2. Geometry of the Layer. In order to derive the equations describing the processes in the layer it is necessary to determine the mean distance between particles l and the minimum traversable section of the layer ψ for a given concentration of particles τ or porosity $\varepsilon = 1 - \tau$, which is the amount of empty space per unit volume of the layer.

It is known [1] that the mean relative traversable area is ε. The minimum relative traversable area ψ, speaking generally, depends on the distribution of particles in the layer. We will examine two limiting cases: a cubic particle lattice, the rarest particle distribution in the layer; and a tetrahedral lattice, one

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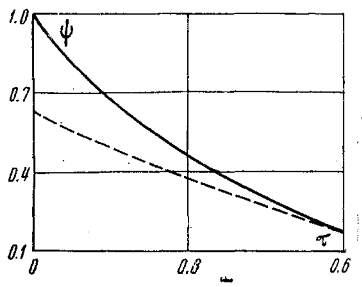

Fig. 1

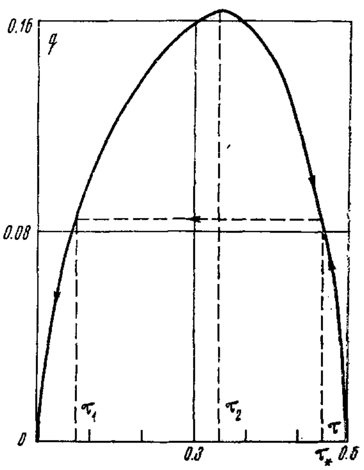

Fig. 2

of the most dense sphere packings possible. We will assume that the real particle distribution is somewhere between these two limiting possibilities.

Simple geometric considerations permit the establishment of the formulas

$$l / d = f(\tau) = (\tau_0/\tau)^{1/3} - 1 \tag{2.1}$$

$$\psi = 1 - (1 - \psi_0)(\tau / \tau_0)^{2/3} \tag{2.2}$$

where d is the particle diameter and l is the interparticle distance.

The values τ_0 and ψ_0 characterize a real layer in a dense state with the same relative particle distribution. We will assume that the value of τ_0 corresponds to a globule concentration in a free packing of random character. According to experimental data, $\tau_0 = 0.6$.

Direct experimental determination of ψ_0 is difficult, and so we shall use the following interpolations, assuming that the real values of τ_0 and ψ_0 may be found from the linear formulas

$$\tau_0 = x\tau_1 + (1 - x)\tau_2, \quad \psi_0 = x\psi_1 + (1 - x)\psi_2 \tag{2.3}$$

Here x is the degree of closeness of the actual packing to cubic, the subscript 1 corresponds to cubical packing; and the subscript 2 to tetrahedral.

$$\tau_0 = \tau_1 \text{ at } x = 1, \quad \tau_0 = \tau_2 \text{ at } x = 0$$

Eliminating x from these expressions and using the numerical values $\tau_1 = 0.524$, $\psi_1 = 0.215$, $\tau_2 = 0.74$, $\psi_2 = 0.096$, we obtain $\psi_0 = 0.17$. Consequently, Eq. (2.2) can be written in the form

$$\psi = 1 - 1.17\tau^{2/3} \tag{2.4}$$

The curve of Eq. (2.4) is presented in Fig. 1. Also shown by a dashed line is the curve of the formula obtained by S. L. Leibenzon [1]

$$\psi = 0.625(1 - \tau)^{1.4}$$

which is obviously of an empirical nature and not applicable for small τ.

3. The Equation of Average Motion and Its Analysis. If the layer of particles is regarded as a gas, then for the mean motion it is possible to write all the dynamic equations of a continuous medium. However, we will limit ourselves to the one-dimensional case and moreover assume that the layer is, on the average, at rest. Then the momentum equation reduces to the equation

$$dq / dy = \tau (F_c - \rho_T g) \tag{3.1}$$

Here the y axis is directed vertically upward, q is the pressure of the particle gas, i.e., the momentum transferred through a unit area in unit time by the particles, ρ_T is the density of the solid phase material, g is the acceleration of gravity, and F_c is the resistance force of a unit volume of the layer.

For the resistance force acting on a single particle, we take the expression

$$f_c = \xi \frac{\pi d^2}{4} \frac{\rho}{2} \left(\frac{v_0}{\psi}\right)^2 \tag{3.2}$$

where ρ is the density of the medium, v_0 is its velocity relative to an empty section, v_0/ψ is the velocity at the minimum traversable section, and ξ is the particle resistance coefficient.

The value of ξ depends on the Reynolds term $Re = v_0 d/\psi \nu$, where ν is the coefficient of kinematic viscosity, but, as the data of [1] shows, is independent of porosity and particle configuration in the layer. Moreover, Eq. (3.2) proves to be applicable to tubular beams with the same value of ξ. Therefore it may be assumed that the function ξ (Re) is universal. With a growth in Re it decreases, but for Re > 1000 ξ stabilizes near a value of 0.5.

Since τ is the fraction of solid phase per unit volume, the number of particles per unit volume will be given by

$$n = 6\tau / \pi d^3 \tag{3.3}$$

Multiplying Eq. (3.3) by Eq. (3.2) and substituting in Eq. (3.1) we obtain the final expression

$$\frac{dq}{dy} = \tau\left(\frac{3}{4}\frac{\xi}{d}\rho\frac{v_0^2}{\psi^2} - \rho_T g\right) = \tau R(\tau) \tag{3.4}$$

In this equation there are two unknowns, τ and q. In order to complete the system it is necessary to know the function $q(\tau)$. To do this, in the following development we will return to the mechanics of the chaotic motion of particles in the layer. However, the general character of the function $q(\tau)$ can be established from Eq. (3.4) alone. Let $v_0 = 0$ initially, then $\tau = \tau_0$, $\psi = \psi_0$ and q(y) is a decreasing linear function, while q(h) = 0, h being the height of the immobile layer. With an increase in v_0 the character of the function q(y) does not change until the equality

$$\frac{3}{4}\frac{\xi}{d}\rho\left(\frac{v_0}{\psi}\right)^2 = \rho_T g$$

is reached.

At that time $q(y) \equiv 0$. With further increase in v_0 the layer begins to boil and the function q(y) becomes positive due to the chaotic particle motion which has begun. Let the value of v_0 be fixed and such that the boiling layer corresponding thereto will be sufficiently thick. We assume that the function $q(\tau)$ in the region of large τ is monotonic on some interval τ. Two cases are possible:

$$\text{a) } \frac{dq}{d\tau} > 0, \quad \text{b) } \frac{dq}{d\tau} < 0$$

We will now write Eq. (3.4) in the form

$$\frac{d\tau}{dy} = \tau\left(\frac{dq}{d\tau}\right)^{-1} R(\tau) \tag{3.5}$$

Considering Eq. (2.4), it is easy to prove that $R(\tau)$ is a monotonic increasing function of τ. We will denote by τ_* that value of τ such that $R(\tau_*) = 0$, assuming that τ_* lies in the chosen interval of large τ. Let the initial value $\tau(0)$, belonging to the same interval, be given for Eq. (3.5). Three cases are possible:

$$\tau(0) > \tau_*, \quad \tau(0) = \tau_*, \quad \tau(0) < \tau_*$$

Let case a) be realized, then for $\tau(0) > \tau_*$ $R(\tau) > 0$ and according to Eq. (3.5) $d\tau/dy > 0$. Thus the layer will condense itself in height. For the case $\tau(0) < \tau_*$ the layer will rarify. Thus case a) is unstable. In case b) any change in τ (0) from τ_* will tend to a reduction, i.e., the concentration in this case will tend to become constant over the height of the layer. It is just this type of behavior that is characteristic of a real layer, and thus it can be concluded that in the region of large τ the function $q(\tau)$ is decreasing. On the other hand, at low concentration levels the behavior of the particle gas should be the same as that of a usual molecular gas, for which under normal conditions pressure increases with an increase in density. Thus over its entire range the function $q(\tau)$ must be nonmonotonic, having a maximum.

4. Equation of State of a Dense Layer. We will examine a unit area located, for example, on the wall. Let the mass of particle be $m = 1/6\ \pi d^3 \rho_T$ and its chaotic velocity at the moment of impact on the area be c. In elementary kinetic theory it is assumed that in a unit time the area is reached by all particles located in a parallelepiped with height c. It is further assumed that the particles are points and screening effects are not considered.

Such assumptions are impermissible for a dense layer. In fact, with the aid of Eq. (2.1) we will estimate the concentration value at which the interparticle distance is less than the diameter. This estimate gives $\tau > 0.075$ ($\varepsilon < 0.925$). Thus the inequality $l/d < 1$ is valid even for quite rarefied layers, so that in a dense layer complete screening occurs, i.e., in the process of collision with the wall only one layer of particles participates, that one directly adjoining the area. The mean number of particles in this layer is $4\tau/\pi d^2$. The momentum transferred to the wall by a single particle is 2mc. Over a time t between collisions the particle traverses a distance $2l$, consequently the time $t = 2l/c$. Thus one particle in unit time transfers to the wall a momentum of mc^2/l, and all particles located next to the area exert a pressure thereon of

$$q = {}^2/_3 \rho_T c^2 \tau / f(\tau) \tag{4.1}$$

where $f(\tau)$ is given by Eq. (2.1).

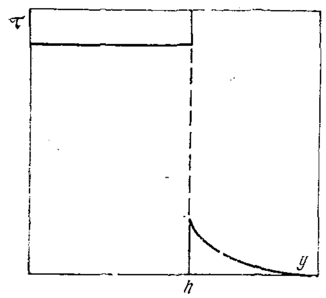

Fig. 3

Equation (4.1) is the sought for equation of state. In it occurs the unknown chaotic particle velocity c, whose determination is the main difficulty of the entire theory. The basic complication lies in explaining the mechanism whereby energy is transferred from the initial flow to the chaotic particle motion, i.e., in setting up the energy balance equation.

5. An Evaluation of the Forces Acting on a Particle in Chaotic Motion and the Function $q(\tau)$. The hydraulic resistance force acting on a particle due to its chaotic motion is of the order of

$$f_c = {}^1/_8 \xi \pi d^2 uc$$

where u is the relative velocity between particle and gas. It may be assumed that the hydrodynamic interaction forces between particles are of the same order.

Inasmuch as the particles inevitably collide with each other and the walls, they must have a rotation acquired in the collision process. In flowing around the rotating particles there develops a transverse Magnus force, whose value is given by the expression [3]

$$f_M = {}^1/_3 \rho \pi d^3 uw$$

where w is the angular velocity of the particle. To determine w we use the principle of equal distribution of energy over degrees of freedom, which occurs under conditions of static equilibrium and for rough spherical molecules, derived in [4]. In this case the principle leads to an equality of translational and rotational energy for the particle, inasmuch as there are six degrees of freedom.

Thus

$$ {}^1/_2\, mc^2 = {}^1/_2\, I\, w^2$$

where $I = 0.1\ md^2$ is the moment of inertia of the sphere.

From this we find

$$wd = \sqrt{10}\, c \tag{5.1}$$

In consideration of Eq. (5.1) the expression for the Magnus force takes on the form

$$f_M = {}^1/_3 \sqrt{10} \rho \pi d^2 uc \tag{5.2}$$

Comparing this expression with f_c, we find

$$\frac{f_M}{f_c} = \frac{8\sqrt{10}}{3\xi} = 16.9 \quad \text{at} \quad \xi = {}^1/_2$$

Thus we see that the Magnus force exceeds the resistance force by more than one order. Also it acts essentially perpendicularly to the flow and therefore gives the fundamental contribution to the chaotic particle motion.

The above makes it possible to consider the following model of chaotic particle motion in the layer. The initial chaoticization is produced by the hydrodynamic instability of the rest configuration. However, this instability only plays the role of a trigger. As soon as the particles begin to collide with each other the Magnus force acts, fulfilling the function of transferring energy from the flow to the layer.

If we assume $c \ll u$, as is actually the case, the relative velocity and the mean flow velocity in the layer are comparable

$$u \approx v_0/\varepsilon$$

Then the expression for the Magnus force, Eq. (5.2), takes on the form

$$f_M = \frac{\sqrt{10}}{3} \rho \pi d^2 \frac{v_0}{\varepsilon} c$$

The work performed by this force on a free path length, which for a dense layer may be equated with the value l, is

$$A_M = \frac{\sqrt{10}}{3} \rho \pi d^2 \frac{v_0}{\varepsilon} cl \tag{5.3}$$

With the dominant role of the Magnus force this energy can not be dissipated due to the hydraulic resistance forces. Therefore it remains to be assumed that it is lost in inelastic collisions of particles. If the precollision velocity had a value of c, the postcollision velocity becomes kc, where the value of k for a head-on collision coincides with the Newton regeneration coefficient, while in other cases it must be calculated with consideration of the scattering angle, which will not be examined here. Equating the energy loss from inelastic collisions $1/2\ mc^2\ (1-k^2)$ to Eq. (5.3) we obtain

$$c = \frac{4\sqrt{10}}{(1-k^2)}\frac{\rho}{\rho_T}\frac{v_0}{\varepsilon}f(\tau) \tag{5.4}$$

Substituting Eq. (5.4) in Eq. (4.1) we obtain the final expression $q(\tau)$

$$q = \frac{107}{(1-k^2)^2}\frac{\rho}{\rho_T}\rho v_0^2 F(\tau), \qquad F(\tau) = \tau\,\frac{\sqrt[3]{\tau_0/\tau}-1}{(1-\tau)^2} \tag{5.5}$$

The function $F(\tau)$ is shown in Fig. 2. The function $q(\tau)$ differs from $F(\tau)$ only by a scale factor.

This function $q(\tau)$ fulfills all the requirements established in Sec. 3. However, in the range of very small τ the function must be corrected since the relationships obtained are not suitable for strongly rarefied layers.

6. Analysis of Results Obtained. As is evident from Fig. 2, the maximum value of $q(\tau)$ is attained at $\tau = \tau_2 = 0.35$ ($\varepsilon = 0.65$). In the figure τ_1 corresponds to vapor. Inasmuch as it is necessary for stability of a dense layer that the inequality $dq/d\tau < 0$ be fulfilled, such a layer can exist only if $\tau_* > 0.35$. This indicates that the inequality must be satisfied so that

$$v_0 < 0.686\sqrt{\rho_T\rho^{-1}gd} \quad \text{at} \quad \xi = 0.5$$

If we introduce the rotational velocity

$$v_b = \sqrt{\frac{4}{3}\frac{\rho_T}{\rho}\frac{gd}{\xi}}$$

the requirement for the existence of a dense layer may be written in the form

$$0.17 < v_0/v_b < 0.42$$

The left-hand side of this inequality corresponds to dense packing at $\tau_0 = 0.6$ and $\psi_0 = 0.17$.

In order to solve Eq. (3.5) with consideration of Eq. (5.5) an initial condition must be given, for example, in the form $\tau(0)$ as well as a condition at infinity $\tau(\infty) = 0$. Moreover, the total particle mass in the layer must be given

$$M = \rho_T\int_0^\infty \tau\,dy$$

However, instead of M it is more convenient to set the height of the dense column h and uniquely determine M over the height.

If $\tau(0) > 0.35$, the solution of Eq. (3.5) for increasing y will tend to the value τ_* and practically attains this value for a sufficiently thick layer. At $y = h$ the concentration drops discontinuously and a phase transition will be accomplished on the left-hand branch of the curve $q(\tau)$, where $dq/d\tau > 0$. The pressure q at the phase boundary does not suffer a discontinuity. Thereafter, in accordance with Eq. (3.5), the concentration will decrease quickly to zero with increasing y. The course of the process is depicted schematically by the arrows in Fig. 2.

The initial condition determines the properties of the lattice supported. For real lattices with a sharp draft it is evident that $\tau(0) < \tau_*$. The same effect should be produced by lattice vibration. In the ideal case of an "adiabatic" lattice, $dq/dy = 0$ at $y = 0$ and $\tau(0) = \tau_*$. In this case the concentration is constant over the entire height of dense column. An example of the function $\tau(y)$ for this case is shown in Fig. 3. The particle mass in the layer is found from the formula

$$M = \rho_T\left(\tau_* h + \int_h^\infty \tau\,dy\right)$$

The greater the equilibrium concentration τ_* the lower the vapor density above the layer. The concentration $\tau_* = 0.35$ plays a critical role. For $\tau_* < 0.35$ independent of the initial conditions the entire layer will be in the vapor phase with concentration decreasing with height. However, the same situation arises for $\tau_* > 0.35$, if $\tau(0) < 0.35$.

Thus, the proposed theory reflects the processes in a boiling layer, including the liquid–vapor phase transition, in a qualitatively true fashion. Quantitative definitions and comparison with experiment have not been conducted since the basic relationships were obtained only in coarse approximations. They require refinement by statistical methods. This is especially true of the parameter k, which has not been determined in this study.

LITERATURE CITED

1. M. E. Aerov and O. M. Todes, Hydraulic and Thermal Principles in the Operation of Apparatus with Stationary and Boiling Granular Layer [in Russian], Khimiya, Leningrad (1968).
2. V. G. Levich and V. P. Myasnikov, "Kinetic theory of the pseudoliquid state," Khim. Prom-st', No. 6 (1966).
3. M. A. Gol'dshtik, and V. N. Sorokin, "Particle motion in a turbulent chamber," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 6 (1968).
4. S. Chapman and T. Cowling, Mathematical Theory of Nonuniform Gasses, Cambridge Univ. Press (1970).